# OPTIMIZACIÓN MULTIOBJETIVO DEL DISEÑO DE UN CASCO DE SUBMARINO

# *MULTIOBJECTIVE OPTIMIZATION OF A SUBMARINE HULL DESIGN*

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## MOTIVATION

Definition of the hull shape is a primary task for all phases of concept design. A parametric formulation of its geometry favors analyzing the hydrodynamics surrounding the vessel.

Slender-body theory can be used to compute the hydrodynamic effects on the ship by using the actual geometry of the ship and thus it can assess the ship's maneuverability.

The shape of the vessel is related as well to the level of resistance.

One of Colombian Ministry of Defense's purposes in naval science and technology focuses on the development of simulators for training. The achievement of a reliable simulation method may also lead to a design tool.

# INTRODUCTION

## OBJECTIVE

Explore the coupling of a submarine dynamics simulation method, derived from slender-body theory, a hull geometry parametric definition and a resistance model so that it can be applied in an optimization process.

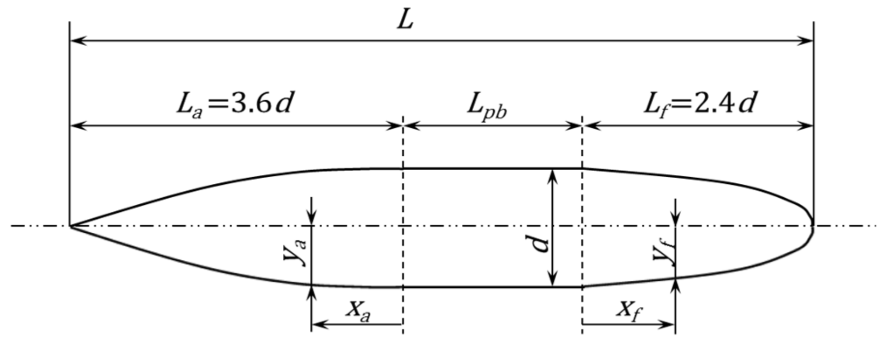
The dynamic model takes into account non-linearities and thereby a numerical simulation is necessary to evaluate the behavior of the vessel in a maneuver.

As a starting point, the report by Zalek & Tascon (2004) on submarine hull optimization (this uses linear maneuverability) is studied and many features of it are kept for the implementation of the herein proposed model.

# SUBMARINE MODEL COMPONENTS

## GEOMETRY

Parametric definition of the hull shape as a body of revolution

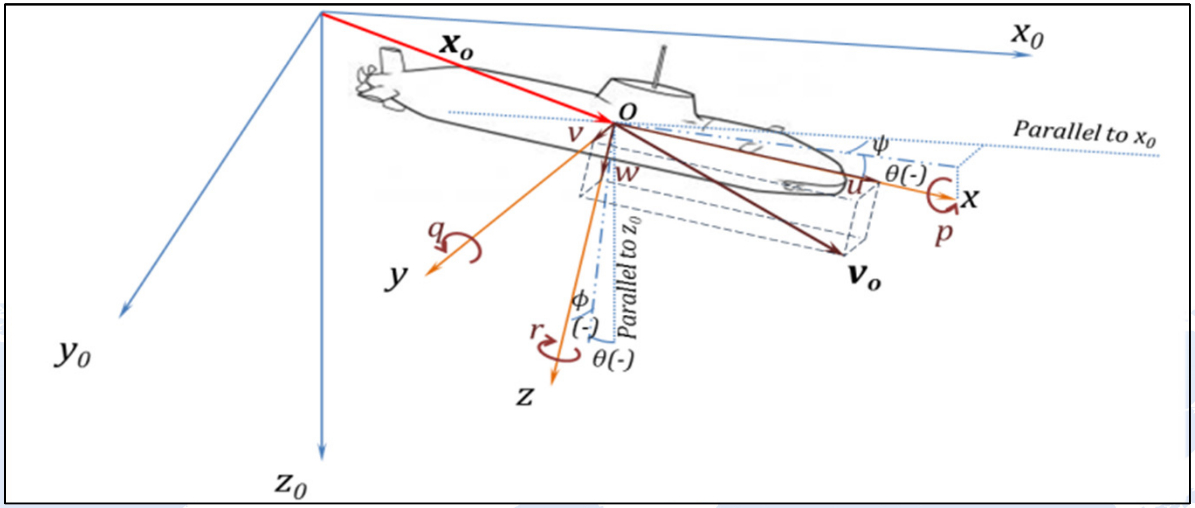



$$y_a = \frac{d}{2}\left[1-\left(\frac{x_a}{L_a}\right)^{n_a}\right]$$

$$y_f = \frac{d}{2}\left[1-\left(\frac{x_f}{L_f}\right)^{n_f}\right]^{1/n_f}$$

# SUBMARINE MODEL COMPONENTS

## MANEUVERABILITY MODEL




Rigid-body dynamics

Six degrees of freedom – Unknowns $v_o = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \omega = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$

Sum of forces acting on the body $\Sigma F = F_H + F_P + F_A$

Forces on the hull $F_H = F_{HD} + F_{HS} + W$

# SUBMARINE MODEL COMPONENTS

## MANEUVERABILITY MODEL

Hydrodynamic forces – Slender body theory

$$\boldsymbol{F}_{\boldsymbol{HD,slender}} = -\int_L \frac{D}{Dt}\left\{\begin{pmatrix} m'_x & 0 & 0 \\ 0 & m'_y & 0 \\ 0 & 0 & m'_z \end{pmatrix}\left[\begin{pmatrix} u \\ v \\ w \end{pmatrix} + \begin{pmatrix} p \\ q \\ r \end{pmatrix} \times \begin{pmatrix} x \\ 0 \\ z_s(x) \end{pmatrix}\right]\right\} dx$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} - u\frac{\partial}{\partial x}$$

Final equations

$$X_{HD} = -m_x\dot{u} + \frac{\rho}{2}L^2 X_{uu}u^2$$

$$\begin{aligned} Y_{HD} = & -m_x ur - \int_L m'_y(\dot{v} + x\dot{r} - z_s(x)\dot{p})dx \\ & - u\left(m'_{ym}(v + x_{ym}r - z_s(x_{ym})p) - \int_{x_a}^{x_{ym}}\left(r - p\frac{\partial z_s(x)}{\partial x}\right)m'_y dx\right) \\ & - \frac{\rho}{2}\int_L h(x)C_{Dy}(v + rx)v_{cross}dx \end{aligned}$$

$$Z_{HD} = m_x uq - \int_L m'_z(\dot{w} - x\dot{q})dx - u\left(m'_{zm}(w - qx_{zm}) - q\int_{x_a}^{x_{zm}} m'_z dx\right) - \frac{\rho}{2}\int_L b(x)C_{Dz}(w - xq)v_{cross}dx$$

# SUBMARINE MODEL COMPONENTS

## MANEUVERABILITY MODEL

### Propulsion force

- $X_P = \rho n^2 D_P^4 (1 - t_P) K_T(J_m)$
- $M_{P,o} = X_P z_P$

### Forces on appendages

- $X_A = X_{rudder} + X_{sail}$
- $Y_A = Y_{rudder} + Y_{sail}$
- $N_A = N_{rudder} + N_{sail}$
- $M_A = M_{sail}$

# SUBMARINE MODEL COMPONENTS

## MANEUVERABILITY MODEL

### Forces on the rudder

$$X_{rudder} = \frac{\rho}{2} L^2 X'_{\delta\delta} \delta^2 u^2$$
$$Y_{rudder} = \frac{\rho}{2} L^2 U^2 Y'_{\delta} \delta$$
$$N_{rudder} = \frac{\rho}{2} L^3 U^2 N'_{\delta} \delta$$

### Forces on the sail

$$X_{sail} = -\frac{\rho}{2} S_{WS} u^2 (C_{fs} + \Delta C_{fs} + C_{rs})$$
$$C_{fs} = \frac{0.075}{(\log Re_S - 2)^2}$$
$$M_{sail} = -X_{sail} h_{SD}$$
$$Y_{sail} = \frac{\rho}{2} (u^2 + v^2) C_{L,sail} A_{sail}$$
$$N_{sail} = Y_A x_{sl}$$

# SUBMARINE MODEL COMPONENTS

## RESISTANCE

- Hull resistance is evaluated by means of a formulation which is related to Reynolds number $Re$, the wetted surface $S_w$ and other geometric parameters ( $d$ and $L$), according to ITTC (1978). The hull resistance plus the contribution from the appendages yield the total resistance of the submarine. This resistance is evaluated at the maximum speed $u_{max}$.

$$R_T = -\frac{\rho}{2} L^2 X_{uu} u_{max}^2 - X_A \Big|_{u=u_{max}}$$

- Hydrodynamic coefficient $X_{uu}$ is defined as follows (Bohlmann, 1991):

$$X_{uu} = -C_f \frac{S_w}{L^2} - \frac{\pi}{4} C_r \frac{d^2}{L^2}$$

$$C_f = \frac{0.075}{(\log Re - 2)^2} + 0.00025$$

# OPTIMIZATION PROBLEM DEFINITION

## OBJECTIVE FUNCTIONS

- *MINIMIZE* Non-dimensional steady turning diameter: $D'_{st}$ (computed through simulation of a turning circle)
- *MINIMIZE* Resistance at the maximum speed $R_T$

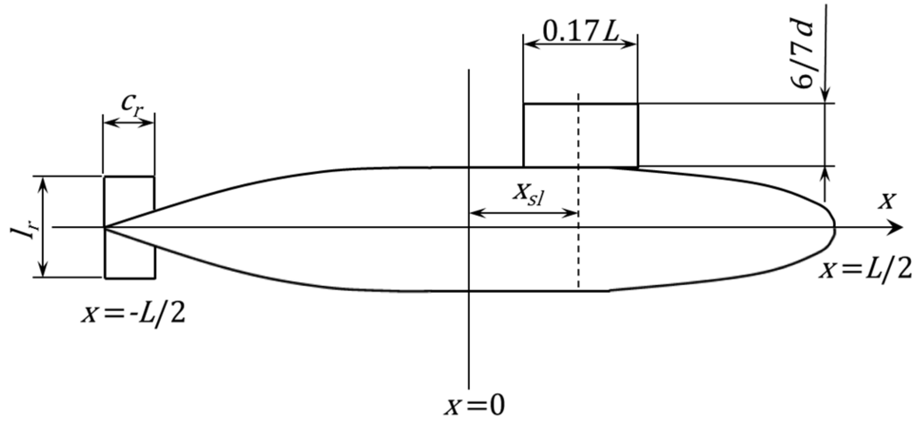
In order to perform the turning circle simulation, the maneuverability model explained above is implemented and a rudder angle is applied.

The resistance is evaluated at the maximum speed, which matches the approach speed of the turning circle, and the vessel is considered to be in pure surge.

# OPTIMIZATION PROBLEM DEFINITION

## DESIGN VARIABLES

| Variable name | Symbol | Lower bound | Upper bound | Unit |
|---|---|---|---|---|
| Hull length | $L$ | 42 | 100 | m |
| Hull diameter | $d$ | 7 | 10 | m |
| Exponent of the radius function for the aft zone | $n_a$ | 1.5 | 5 | - |
| Exponent of the radius function for the forward zone | $n_f$ | 1.5 | 5 | - |
| Sail longitudinal location | $x_{sl}$ | 6.3 | 20 | m |
| Total rudder area | $A_R$ | 6 | 20 | $m^2$ |
| Rudder aspect ratio | $\Lambda$ | 0.6 | 1.3 | - |




$$l_r = \sqrt{A_R \Lambda} \qquad c_r = A_R / l_r$$

# OPTIMIZATION PROBLEM DEFINITION

## CONSTRAINTS

- Parallel middle body length is non-negative:

$$L_{pb} \geq 0 \; \rightarrow \; L \geq 6d$$

- Sail's position is between 15% and 20% of length :

$$0.15L - x_{sl} \leq 0, -0.2L + x_{sl} \leq 0$$

- The rudder area has to be at least equal to a proportion of the product *Ld* (3%), and at the most as a function of the envelope volume $\nabla$:

$$A_R \geq 0.03Ld = A_{R,bot}, A_R \leq 2\left(0.07\nabla^{2/3}\right) = A_{R,top}$$

- The volume has to overtake a minimal capacity $\nabla_{min}$:

$$\nabla \geq \nabla_{min}$$

- The deck area must be greater than or at least equal to a given value $A_{deck,min}$:

$$A_{deck} = 2\int_0^{L_a} y_a dx_a + L_{pb}d + 2\int_0^{L_f} y_f dx_f \geq A_{deck,min}$$

- The rudder must not span beyond the hull diameter:

$$\frac{l_r}{2} \leq \frac{d}{2} - y_r$$

$$y_r = y_a(x_r), x_r = L_a - c_r.$$

# OPTIMIZATION PROBLEM DEFINITION

## PARAMETERS

| Parameter name | Symbol | Value | Source |
|---|---|---|---|
| Centre of gravity | $(x_g, y_g, z_g)$ | (0,0,0) | Current work |
| Centre of buoyancy | $(x_{by}, y_{by}, z_{by})$ | (0,0,0) | Current work |
| Approach speed | $U_0$ | 20 knots | Current work |
| Minimum deck area | $A_{deck,min}$ | 100 $m^2$ | Current work |
| Minimum volume | $\nabla_{min}$ | 800 $m^3$ | Current work |
| Water density | $\rho$ | $1024 kg/m^3$ | Sharqawy et al. (2010) |
| Water viscosity | $\nu$ | $1.05 \times 10^{-6} m^2/s$ | Sharqawy et al. (2010) |
| Residual hull resistance coefficient | $C_r$ | 0.013 | Jackson (1992) |
| Residual sail resistance coefficient | $C_{rS}$ | 0.005 | Zalek & Tascon (2004) |
| Sail roughness coefficient | $\Delta C_{fS}$ | 0.0004 | Zalek & Tascon (2004) |
| Centre of pressure of sail's drag | $h_{SD}$ | $38d/21\pi$ | Watt (2007) |
| Propeller diameter | $D_P$ | $d/2$ | Current work |
| Sail length | $L_{sl}$ | $0.17L$ | Current work |
| Sail height | $h_{sl}$ | $6d/7$ | Current work |

# RESULTS

## Optimization run by a genetic algorithm plugin in ModelCenter®

**Population size per generation**: 100 designs

**Maximum number of generations**: 100

Two cases, varying rudder angle:

**For δ=20°**

Solutions in Pareto front: 16, after 65 generations

**For δ=30°**

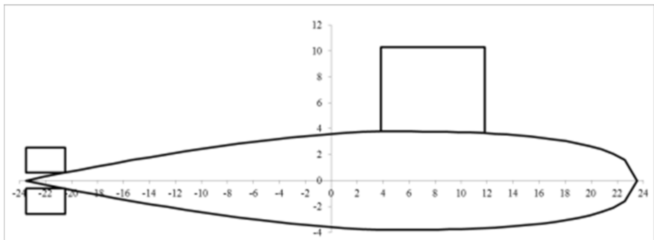
Solutions in Pareto front: 11, after 100 generations

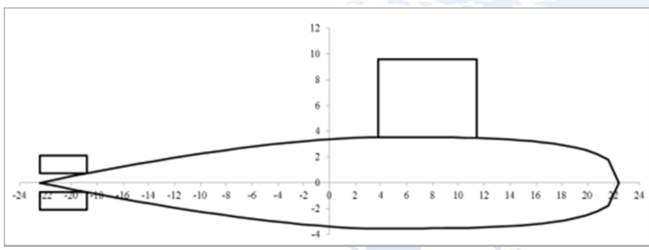
# RESULTS

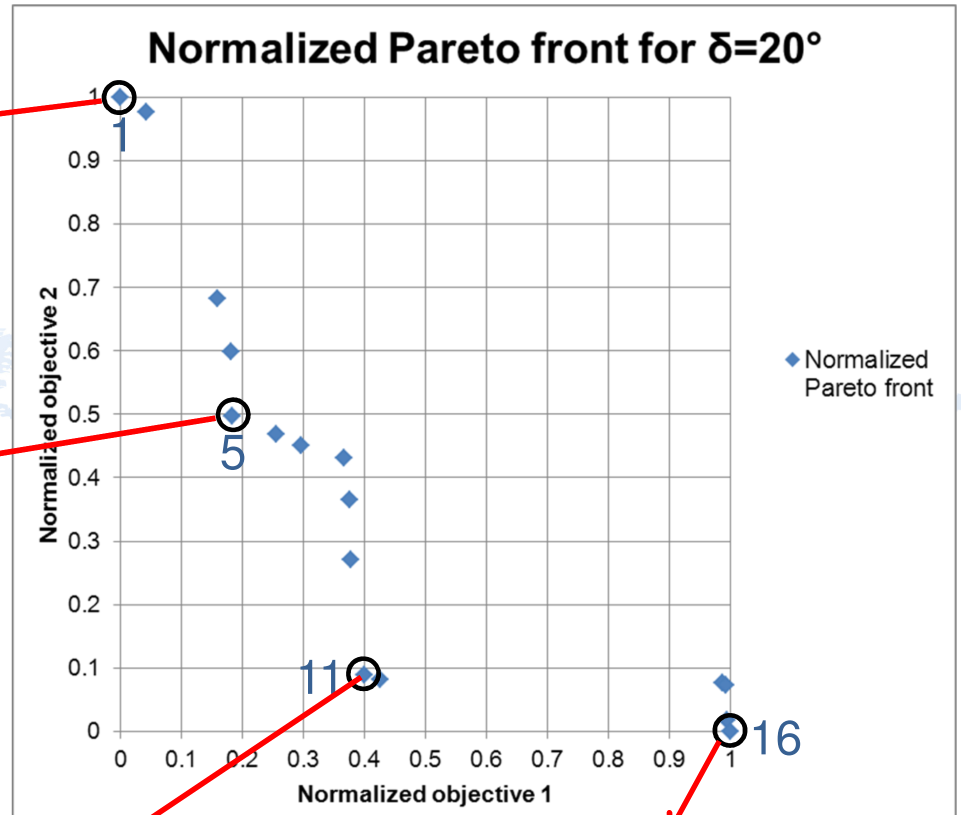
## For δ=20°

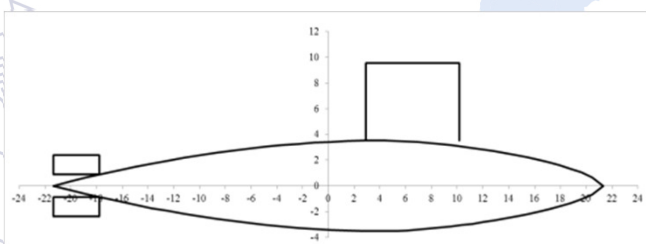
| | | Design variables | | | | | | | Objectives | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $L$ | $d$ | $n_a$ | $n_f$ | $x_{sl}$ | $A_R$ | $\Lambda$ | $D'_{st}$ | $R_T$ |
| | Unit | m | m | - | - | m | $m^2$ | - | - | N |
| PARETO FRONT DESIGNS | 1 | 47.014 | 7.569 | 1.514 | 2.465 | 7.797 | 11.710 | 1.291 | 0.805 | 141987 |
| | 2 | 46.418 | 7.637 | 1.516 | 2.533 | 7.668 | 11.880 | 1.295 | 0.820 | 141364 |
| | 3 | 45.307 | 7.293 | 1.611 | 2.608 | 7.646 | 11.710 | 0.663 | 0.860 | 133541 |
| | 4 | 45.088 | 7.195 | 1.699 | 2.409 | 7.568 | 11.410 | 0.620 | 0.868 | 131343 |
| | 5 | 44.873 | 7.072 | 1.501 | 2.904 | 7.607 | 10.070 | 0.754 | 0.868 | 128642 |
| | 6 | 44.568 | 7.169 | 1.692 | 2.096 | 7.473 | 12.240 | 0.772 | 0.893 | 127893 |
| | 7 | 44.479 | 7.078 | 1.652 | 2.443 | 7.550 | 12.090 | 0.848 | 0.907 | 127421 |
| | 8 | 44.674 | 7.116 | 1.670 | 2.027 | 7.575 | 12.590 | 0.947 | 0.931 | 126911 |
| | 9 | 43.523 | 7.093 | 1.648 | 2.588 | 7.365 | 13.010 | 0.873 | 0.934 | 125154 |
| | 10 | 43.363 | 7.119 | 1.832 | 1.797 | 7.265 | 12.440 | 0.788 | 0.935 | 122669 |
| | 11 | 42.652 | 7.038 | 1.889 | 1.554 | 6.551 | 10.650 | 0.836 | 0.943 | 117829 |
| | 12 | 42.804 | 7.000 | 1.913 | 1.521 | 6.782 | 10.610 | 0.829 | 0.952 | 117631 |
| | 13 | 42.088 | 7.005 | 1.848 | 1.836 | 8.166 | 12.890 | 0.791 | 1.144 | 117499 |
| | 14 | 42.000 | 7.007 | 1.881 | 1.824 | 7.693 | 12.570 | 0.797 | 1.146 | 117403 |
| | 15 | 42.078 | 7.007 | 1.836 | 1.627 | 8.107 | 12.320 | 0.803 | 1.147 | 115937 |
| | 16 | 42.000 | 7.000 | 1.844 | 1.605 | 8.038 | 12.420 | 0.805 | 1.149 | 115472 |

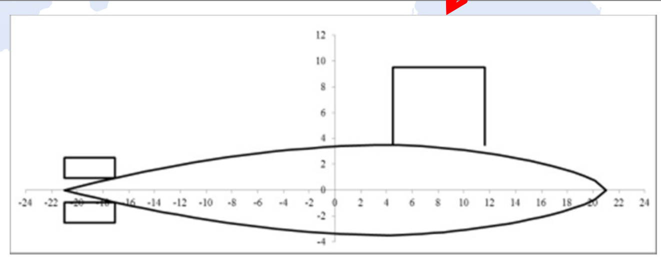
# RESULTS






Normalized Pareto front for δ=20°
Normalized objective 2
Normalized objective 1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized Pareto front
1
5
11
16



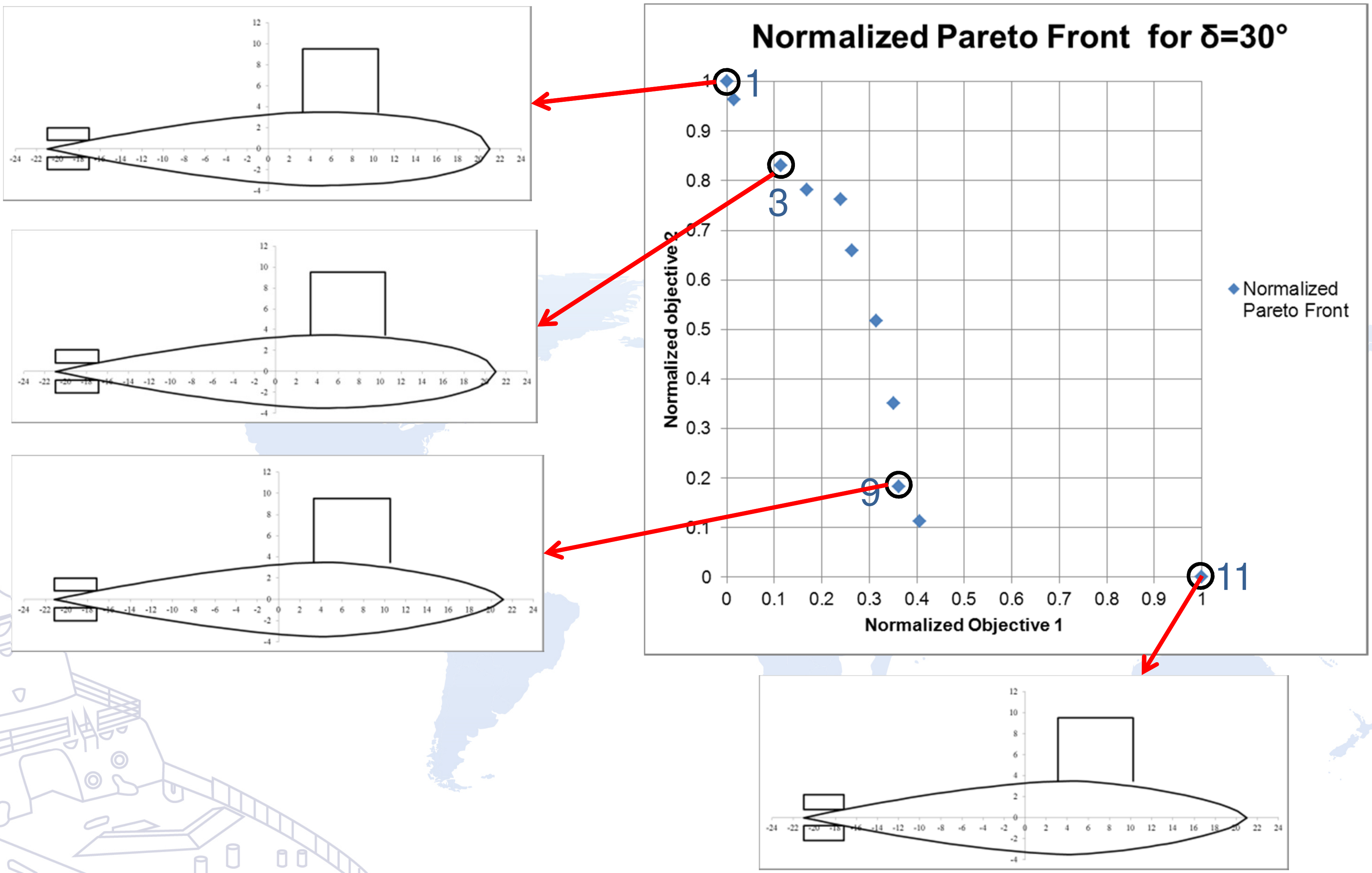


# RESULTS

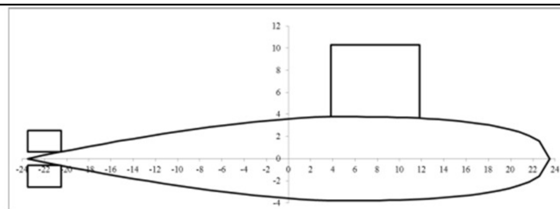
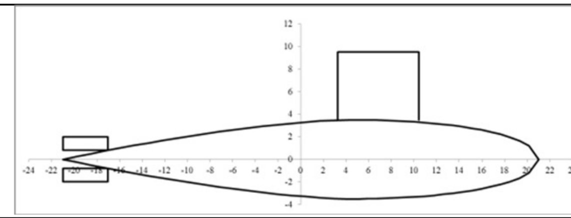
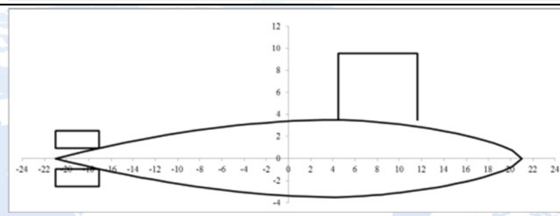
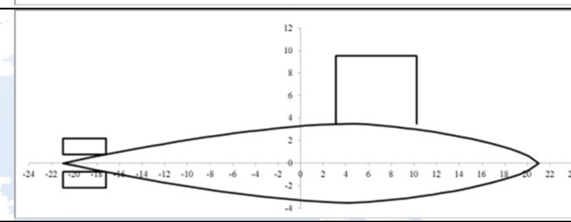
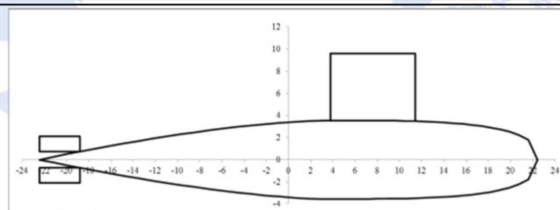
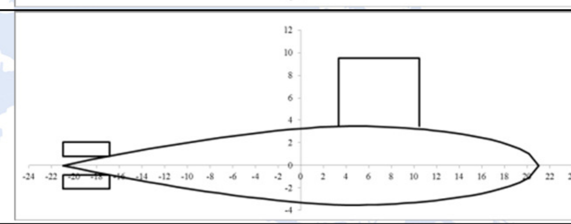
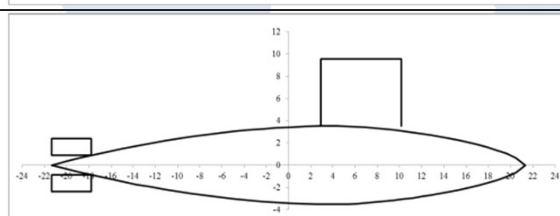
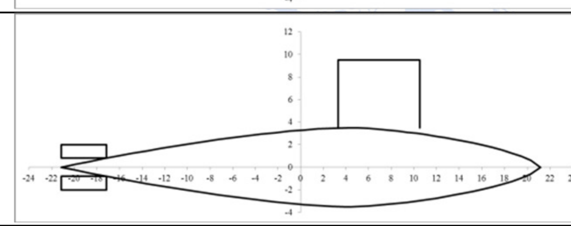
## For δ=30°

| | | Design variables | | | | | | | Objectives | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $L$ | $d$ | $n_a$ | $n_f$ | $x_{sl}$ | $A_R$ | $\Lambda$ | $D'_{st}$ | $R_T$ |
| | Unit | m | m | - | - | m | $m^2$ | - | - | N |
| PARETO FRONT DESIGNS | 1 | 42 | 7 | 1.5 | 2.134 | 6.854 | 9.51 | 0.6 | 0.810 | 116220 |
| | 2 | 42 | 7 | 1.5 | 2.104 | 6.757 | 9.57 | 0.6 | 0.811 | 116079 |
| | 3 | 42 | 7.003 | 1.517 | 1.968 | 6.912 | 10.13 | 0.6 | 0.815 | 115570 |
| | 4 | 42.100 | 7 | 1.512 | 1.897 | 6.860 | 10.01 | 0.601 | 0.818 | 115380 |
| | 5 | 42 | 7.003 | 1.553 | 1.872 | 6.741 | 10.11 | 0.6 | 0.821 | 115308 |
| | 6 | 42.026 | 7.003 | 1.509 | 1.856 | 6.842 | 10.22 | 0.628 | 0.822 | 114909 |
| | 7 | 42.075 | 7 | 1.5 | 1.766 | 6.807 | 10.3 | 0.630 | 0.824 | 114367 |
| | 8 | 42 | 7 | 1.515 | 1.690 | 6.803 | 10.26 | 0.629 | 0.826 | 113728 |
| | 9 | 42.306 | 7.006 | 1.505 | 1.5 | 6.910 | 9.66 | 0.6 | 0.826 | 113078 |
| | 10 | 42.238 | 7.001 | 1.505 | 1.5 | 6.846 | 10.5 | 0.6 | 0.828 | 112810 |
| | 11 | 42 | 7 | 1.535 | 1.506 | 6.687 | 10.65 | 0.736 | 0.854 | 112377 |

# RESULTS


Normalized Pareto Front for δ=30°
1
3
9
11
Normalized objective 2
Normalized Objective 1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized Pareto Front

# RESULTS

| Main characteristic | Obtained shape by optimization with δ=20° | Obtained shape by optimization with δ=30° |
|---|---|---|
| Minimal turning diameter |  |  |
| Minimal resistance |  |  |
| 1st compromise solution |  |  |
| 2nd compromise solution |  |  |

# DISCUSSION

For a minimal turning diameter a bulkier forward zone of the hull is seen. Besides, a very slim aft zone is noticed in all designs. On the other hand, if the resistance is the minimization objective, a slim forward body is obtained.

The integrals that compose the hydrodynamic forces equations derived from the slender body theory are mainly evaluated between the section of greatest added mass and the bow. Since the aft body shape has a smaller effect on the maneuvering coefficients, the forward part of the hull is the one which changes the most in order to optimize the design.

In most cases the length and hull diameter were at their lower bounds. Since these variables are almost constant for every obtained solution, and the aft zone is thin, the shape of the forward body affects the wetted surface and volume of them. This fact therefore influences the resulting resistance.

# DISCUSSION

## Active constraints

### For δ=20°

| Design | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $L_{pb}$(m) | 1.60 | 0.60 | 1.55 | 1.91 | 2.44 | 1.55 | 2.01 | 1.98 | 0.96 | 0.65 | 0.43 | 0.80 | 0.06 | -0.04 | 0.04 | 0.00 |
| $x_{st}/L$ | 0.17 | 0.17 | 0.17 | 0.17 | 0.17 | 0.17 | 0.17 | 0.17 | 0.17 | 0.17 | 0.15 | 0.16 | 0.19 | 0.18 | 0.19 | 0.19 |
| (m$^3$) | 1236 | 1231 | 1139 | 1107 | 1077 | 1044 | 1054 | 1025 | 1032 | 966 | 890 | 887 | 907 | 907 | 870 | 862 |
| $A_{deck}$ (m$^2$) | 210 | 207 | 194 | 187 | 194 | 178 | 183 | 178 | 178 | 160 | 147 | 147 | 151 | 150 | 147 | 145 |
| $A_R$-$A_{R,bot}$ (m$^2$) | 1.03 | 1.25 | 1.80 | 1.68 | 0.55 | 2.65 | 2.64 | 3.05 | 3.75 | 3.18 | 1.64 | 1.62 | 4.05 | 3.74 | 3.47 | 3.60 |
| $A_R$-$A_{R,top}$ (m$^2$) | -4.42 | -4.20 | -3.57 | -3.58 | -4.64 | -2.17 | -2.42 | -1.65 | -1.29 | -1.25 | -2.31 | -2.32 | -0.23 | -0.55 | -0.44 | -0.27 |
| $l_r/2$-$(d/2$-$y_r)$ (m) | -1.23 | -1.24 | -1.36 | -1.32 | -1.42 | -1.16 | -1.11 | -1.03 | -1.02 | -1.05 | -1.15 | -1.13 | -0.94 | -0.96 | -1.00 | -0.98 |

### For δ=30°

| Design | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $L_{pb}$(m) | 0 | 0 | -0.02 | 0.100 | -0.02 | 0.011 | 0.075 | 0 | 0.271 | 0.229 | 0 |
| $x_{st}/L$ | 0.163 | 0.161 | 0.165 | 0.163 | 0.161 | 0.163 | 0.162 | 0.162 | 0.163 | 0.162 | 0.159 |
| (m$^3$) | 884 | 881 | 867 | 859 | 860 | 850 | 836 | 823 | 797 | 794 | 792 |
| $A_{deck}$ (m$^2$) | 165 | 165 | 162 | 161 | 159 | 160 | 159 | 156 | 153 | 153 | 150 |
| $A_R$-$A_{R,bot}$ (m$^2$) | 0.69 | 0.75 | 1.31 | 1.17 | 1.29 | 1.39 | 1.46 | 1.44 | 0.77 | 1.63 | 1.83 |
| $A_R$-$A_{R,top}$ (m$^2$) | -3.39 | -3.29 | -2.60 | -2.64 | -2.55 | -2.34 | -2.12 | -2.04 | -2.38 | -1.51 | -1.33 |
| $l_r/2$-$(d/2$-$y_r)$ (m) | -1.51 | -1.50 | -1.44 | -1.45 | -1.43 | -1.42 | -1.42 | -1.42 | -1.49 | -1.41 | -1.32 |

# DISCUSSION

Three suggested alternatives to reduce the submarine drag consist of increasing length, reducing the wetted surface or increasing the length to diameter ratio.

The length in all designs are about its lower bound so that the first alternative cannot be proven here.

Resistance is minimized with a decreasing wetted surface so the second alternative is verified.

Wetted surface has a major influence on the ship's resistance so the optimization process first tends to get to solutions with a more reduced length and finally makes wetted surface decrease by modifying the shape exponents.

The third statement is proven as the highest length-hull ratio is attained with the smallest hull diameter.

# CONCLUSIONS

- Resulting designs showed consistency regarding the relationship between wetted surface and resistance. Minimal required capacity is fairly obtained in the solutions of minimal resistance and widely accomplished in the solutions with the smallest turning diameters.
- The way that the optimization enhances the turning ability lies on the variation of the forward zone of the submarine hull because this part of the body is the one that affects the hydrodynamic derivatives the most, and therefore the vessel's maneuverability.
- Some design considerations were observed to check if the obtained solutions agreed with them. It was provided a possible explanation on how the optimization process led to the final set of designs.
- There was a graphical identification of a compromise but the data provided can be used by the decision-maker if a new criterion is preferred.

# CONCLUSIONS

## FUTURE WORK

- Implementation of a more complete model and other maneuvers is sought so that more reliable resulting designs can be obtained. That model could enhance the way in which appendages and propulsion forces are assessed, though the number of variables may increase if there's a higher complexity of the parametric geometry.
- Consider interactions between the rudder and the hull and other effects so that a set of formulae for the hydrodynamic coefficients can be achieved. With those coefficients, an analysis of stability and maneuverability can be performed, which can be incorporated to the optimization model as a new criterion or constraint.

# GRACIAS POR SU ATENCIÓN

# *THANKS FOR YOUR ATTENTION*